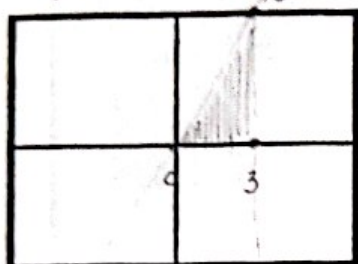


Key

Example 1) Find the volume if the region enclosing $y = 2x, y = 0, x = 3$ is rotated about the

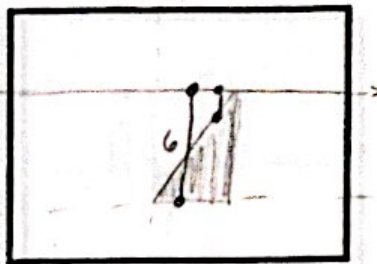
a) x-axis



$$R = \frac{2x}{r} \quad r = \frac{6}{x}$$

$$V = \pi \int_0^3 (2x)^2 dx = 36\pi$$

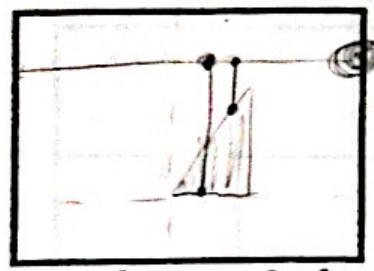
b) the line $y = 6$



$$R = \frac{6}{r} \quad r = \frac{6-2x}{x}$$

$$V = \pi \int_0^3 (6^2 - (6-2x)^2) dx = 72\pi$$

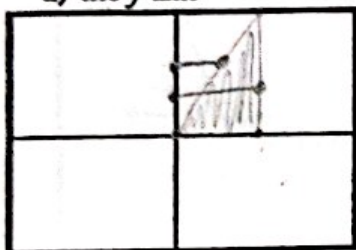
c) the line $y = 8$



$$R = \frac{8}{r} \quad r = \frac{8-2x}{x}$$

$$V = \pi \int_0^3 (8^2 - (8-2x)^2) dx = 108\pi$$

d) the y-axis



$$R = \frac{3}{r} \quad r = \frac{1/2}{x}$$

$$V = \pi \int_0^6 (3^2 - (\frac{x}{2})^2) dy = 36\pi$$

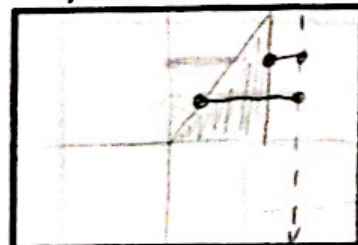
e) the line $x = 3$



$$R = \frac{3 - 1/2}{r} \quad r = \frac{1}{x}$$

$$V = \pi \int_0^6 (3 - \frac{1}{2})^2 dy = 18\pi$$

f) the line $x = 4$

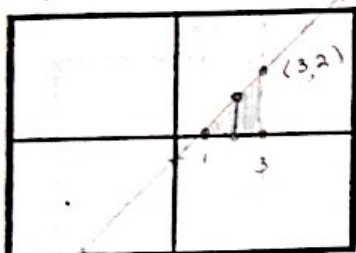


$$R = \frac{4 - 1/2}{r} \quad r = \frac{1}{x}$$

$$V = \pi \int_0^6 [(4 - \frac{1}{2})^2 - 1^2] dy = 36\pi$$

Example 2) Find the volume if the region enclosing $y = x - 1, y = 0, x = 3$ is rotated about the

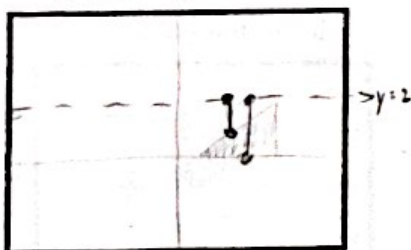
a) x-axis



$$R = \frac{x-1}{r} \quad r = \frac{2}{x}$$

$$V = \pi \int_1^3 (x-1)^2 dx = \frac{8\pi}{3}$$

b) the line $y = 2$



$$R = \frac{2}{r} \quad r = \frac{2-(x-1)}{x}$$

$$V = \pi \int_1^3 [4 - (3-x)^2] dx = \frac{16\pi}{3}$$

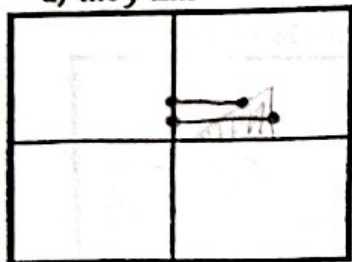
c) the line $y = 3$



$$R = \frac{3}{r} \quad r = \frac{3-(x-1)}{x}$$

$$V = \pi \int_1^3 [9 - (4-x)^2] dx = \frac{28\pi}{3}$$

d) the y-axis



$$R = \frac{3}{r} \quad r = \frac{y+1}{x}$$

$$V = \pi \int_2^3 [9 - (y+1)^2] dy = \frac{28\pi}{3}$$

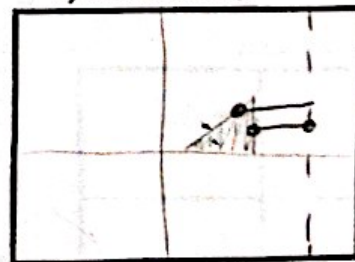
e) the line $x = 3$



$$R = \frac{3-(y+1)}{r} \quad r = \frac{1}{x}$$

$$V = \pi \int_2^3 (2-y)^2 dy = \frac{8\pi}{3}$$

f) the line $x = 5$

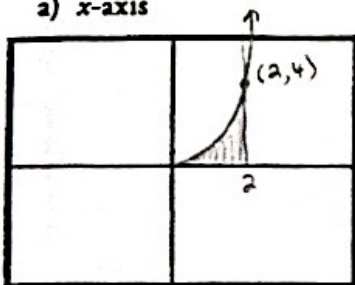


$$R = \frac{5-(y+1)}{r} \quad r = \frac{2}{x}$$

$$V = \pi \int_2^3 [(4-y)^2 - 4] dy = \frac{32\pi}{3}$$

Example 3) Find the volume if the region enclosing $y = x^2, y = 0, x = 2$ is rotated about the

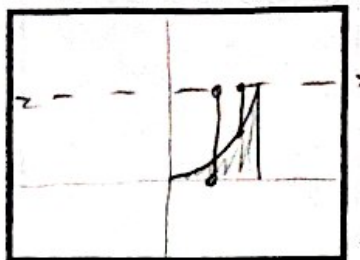
a) x-axis



$$R = x^2 \quad r = \frac{1}{2}$$

$$V = \pi \int_0^2 (x^2)^2 dx = 32\pi/5$$

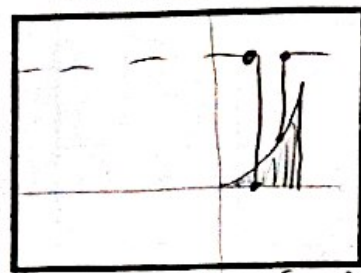
b) the line $y = 4$



$$R = 4 \quad r = 4 - x^2$$

$$V = \pi \int_0^2 [16 - (4 - x^2)^2] dx = 224\pi/5$$

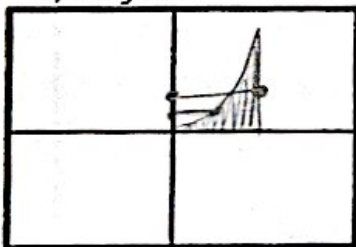
c) the line $y = 5$



$$R = 5 \quad r = 5 - x^2$$

$$V = \pi \int_0^2 [25 - (5 - x^2)^2] dx = 304\pi/5$$

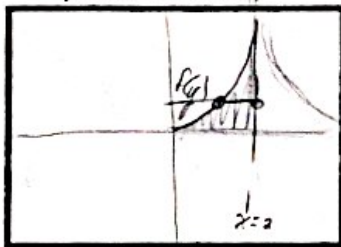
d) the y-axis



$$R = 2 \quad r = \sqrt{y}$$

$$V = \pi \int_0^4 [4 - y] dy = 8\pi$$

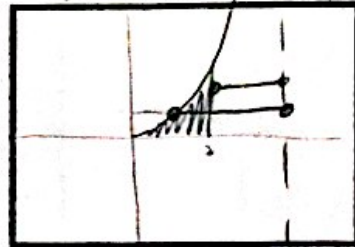
e) the line $x = 2$



$$R = 2 - \sqrt{y} \quad r = \frac{1}{2}$$

$$V = \pi \int_0^4 (2 - \sqrt{y})^2 dy = 8\pi/3$$

f) the line $x = 4$

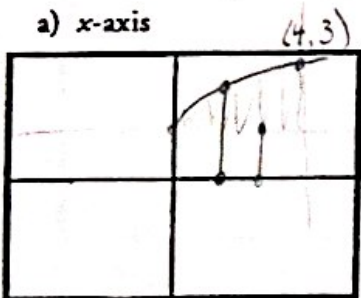


$$R = 4 - \sqrt{y} \quad r = \frac{1}{2}$$

$$V = \pi \int_0^4 [(4 - \sqrt{y})^2 - 4] dy = 40\pi/3$$

Example 4) Find the volume if the region enclosing $y = 1 + \sqrt{x}, y = 1, x = 4$ is rotated about the

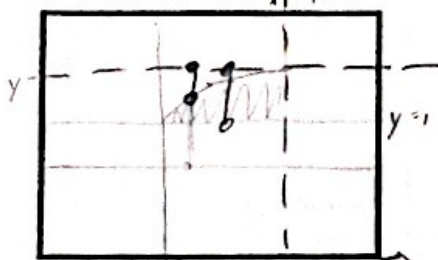
a) x-axis



$$R = 1 + \sqrt{x} \quad r = 1$$

$$V = \pi \int_0^4 [(1 + \sqrt{x})^2 - 1] dx = 56\pi/3$$

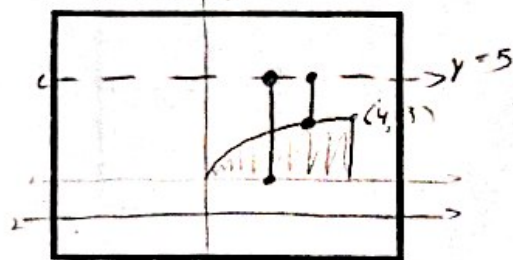
b) the line $y = 3$



$$R = 3 \quad r = 3 - (1 + \sqrt{x})$$

$$V = \pi \int_0^4 [4 - (2 + \sqrt{x})^2] dx = 40\pi/3$$

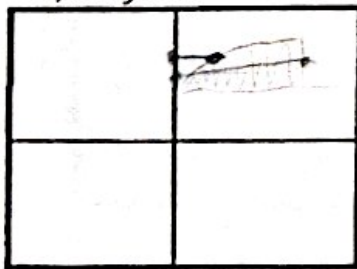
c) the line $y = 5$



$$R = 5 \quad r = 5 - (1 + \sqrt{x})$$

$$V = \pi \int_0^4 [16 - (4 + \sqrt{x})^2] dx = 104\pi/3$$

d) the y-axis



$$R = 4 \quad r = (\sqrt{y} - 1)^2$$

$$V = \pi \int_1^5 [16 - (\sqrt{y} - 1)^2] dy = 128\pi/3$$

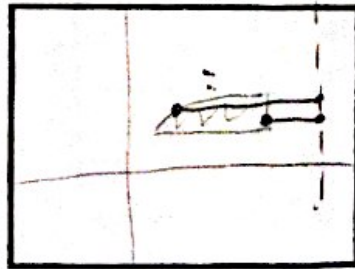
e) the line $x = 4$



$$R = 4 - (\sqrt{y} - 1)^2 \quad r = \frac{1}{2}$$

$$V = \pi \int_1^5 [4 - (\sqrt{y} - 1)^2] dy = 256\pi/15$$

f) the line $x = 6$

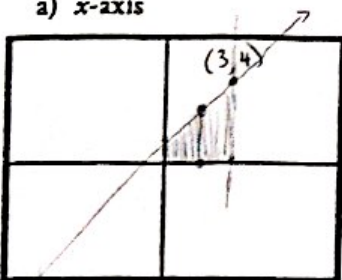


$$R = 6 - (\sqrt{y} - 1)^2 \quad r = \frac{1}{2}$$

$$V = \pi \int_1^5 [(6 - (\sqrt{y} - 1)^2)^2 - 4] dy = 192\pi/5$$

Example 5) Find the volume if the region enclosing $y = x + 1, x = 0, y = 0, x = 3$ is rotated about the

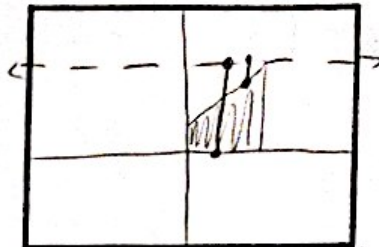
a) x-axis



$$R = x+1 \quad r = 0$$

$$V = \pi \int_0^3 (x+1)^2 dx = 21\pi$$

b) the line $y = 4$



$$R = 4 \quad r = 4 - (x+1)$$

$$V = \pi \int_0^3 [16 - (3-x)^2] dx = 39\pi$$

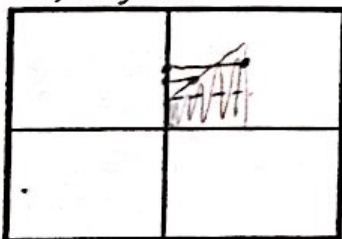
c) the line $y = 5$



$$R = 5 \quad r = 5 - (x+1)$$

$$V = \pi \int_0^3 [25 - (4-x)^2] dx = 54\pi$$

d) the y-axis



$$R = 3 \quad r = y-1$$

$$V = 9\pi + \pi \int_1^4 [9 - (y-1)^2] dy = 27\pi$$

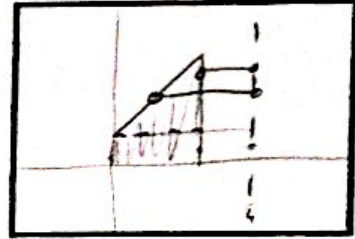
e) the line $x = 3$



$$R = 3 - (y-1) \quad r = 0$$

$$V = 9\pi + \pi \int_1^4 (4-y)^2 dy = 18\pi$$

f) the line $x = 4$

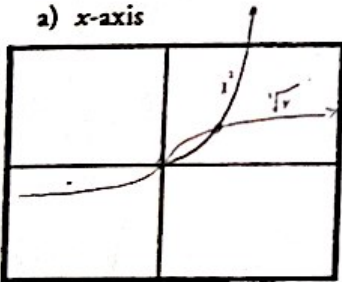


$$R = 4 - (y-1) \quad r = 1$$

$$V = 15\pi + \pi \int_1^4 [(5-y)^2 - 1] dy = 33\pi$$

Example 6) Find the volume if the region enclosing $y = x^2, y = \sqrt[3]{x}$ is rotated about the

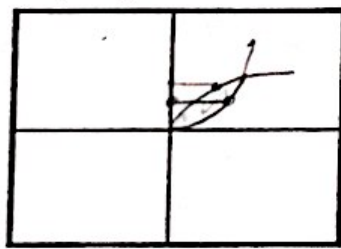
a) x-axis



$$R = \sqrt[3]{x} \quad r = x^2$$

$$V = \pi \int_0^1 (x^{2/3} - x^4) dx = 2\pi/5$$

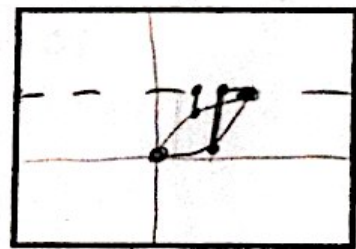
b) the y-axis



$$R = \sqrt{y} \quad r = y^{3/4}$$

$$V = \pi \int_0^1 (y - y^6) dy = 5\pi/14$$

c) the line $y = 1$



$$R = 1 - x^2 \quad r = 1 - \sqrt[3]{x}$$

$$V = \pi \int_0^1 [(1-x^2)^2 - (1-\sqrt[3]{x})^2] dx = \frac{13\pi}{30}$$

∴